



# Local thermal nonequilibrium effects in forced convection in a porous medium channel: a conjugate problem

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## Abstract

Forced convection in a plane channel filled with a saturated porous medium, coupled with conduction in plane slabs bounding the channel, is investigated analytically on the basis of a two-temperature model allowing for local thermal nonequilibrium (LTNE). It is found that the effect of the finite thermal resistance due to the slabs is to reduce both the heat transfer to the porous medium and the degree of LTNE. An increase in value of the Péclet number leads to a decrease in the rate of exponential decay in the downstream direction but does not affect the value of a suitably defined Nusselt number. The dependence of the Nusselt number on a new solid–fluid heat exchange parameter, the solid/fluid thermal conductivity ratio, and the porosity, is investigated. The general two-temperature formulation of the thermal boundary conditions is discussed. © 1999 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

There are several industrial applications where high speed flow in a saturated porous medium leads to a significant degree of local thermal nonequilibrium (LTNE). One example is fixed bed nuclear propulsion systems and nuclear reactor modeling where the temperature difference between the liquid coolant and the solid rods is of crucial importance. A second example is the storage of thermal energy derived from a solar energy conversion system, where a heated fluid flows from the solar collectors into a bed of rocks [1,2], and energy is recovered by reversing the flow in the bed. Other storage systems have been designed for space power supply systems [3]. In some storage systems,

phase change material is used to enhance the efficiency [4].

Most theoretical and numerical work on LTNE, dating from the classical paper of [5], has been concerned with transient situations. Examples are the recent studies of forced convection, by [6–8], and the various papers by Kuznetsov; see e.g. [9] and the review [10]. Exceptions are the numerical study of a steady state situation by [11] and the analytical study by [12]; these are concerned with a steady-state situation. The latter author explicitly investigated the circumstances in which LTNE was significant in general steady processes, even in the absence of longitudinal dispersion.

The present study is essentially an extension of the [12] work on steady forced convection in a channel between plane parallel walls. The extension is to a conjugate problem, involving the coupling of convection in the porous medium channel with conduction in

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### Nomenclature

$A, B$	constants appearing in Eq. (26)
$Bi$	Biot number
$c_p$	specific heat at constant pressure
$Da$	Darcy number
$h$	wall heat transfer coefficient
$h_{fs}$	fluid–solid heat transfer coefficient
$H$	channel half-spacing
$H'$	boundary slab thickness
$k$	thermal conductivity
$k'$	boundary slab thermal conductivity
$k_r$	thermal conductivity ratio, $k_s/k_f$
$K$	permeability
$L_f, L_s$	fluid and solid Biot numbers
$N_h, N_f, N_s$	parameters defined in Eq. (13)
$Nu$	Nusselt number
$Pe$	Péclet number
$q''$	wall heat flux
$s_1, s_2$	quantities defined in Eqs. (27) and (28)
$T$	temperature
$T_0$	outside temperature
$U$	Darcy velocity in channel
$\mathbf{v}$	Darcy velocity
$x, y$	spatial coordinates.

### Greek symbols

$\beta$	constant defined in Eq. (1)
$\eta$	fluid–solid heat exchange parameter
$\theta$	temperature difference
$\Theta$	temperature difference amplitude
$\lambda$	exponent defined in Eq. (19)
$\rho$	density
$\phi$	porosity.

### Subscripts

b	bulk
eff	effective
f	fluid
ref	reference
s	solid
w	wall.

adjacent solid slabs. The geometry of the problem is illustrated in Fig. 1. We consider a porous medium channel of half width  $H$  bounded on each side by a boundary solid slab of thickness  $H'$ . To the best of our knowledge, the effect of LTNE in a conjugate conduction–convection situation has not been previously examined. The extension obviously has a practical application to situations where the wall presents a finite thermal resistance to heat transfer from the environment to the porous medium.

We are also motivated by a desire to clarify the nature of the thermal boundary conditions at the boundary of the porous medium. It is not sufficient merely to satisfy the requirement of continuity of temperature and the heat flux on the scale of a representative elementary volume (REV) at the boundary. The full definition of the problem requires another boundary condition. In two limiting cases it is clear what the second condition must be. If the boundary is perfectly conducting, so that the temperature  $T_w$  of the wall is

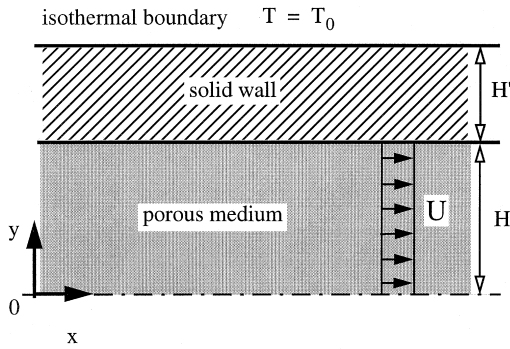


Fig. 1. Definition sketch.

constant, then it is obvious that the fluid phase temperature  $T_f$  and the solid phase temperature  $T_s$  should satisfy  $T_f = T_w$  and  $T_s = T_w$ , and so  $T_f = T_s$  at the boundary, i.e. there is thermal equilibrium there. Further, if the boundary is thermally insulating, then the normal derivative of both  $T_f$  and  $T_s$  vanishes there. In a more general case, e.g. when nonzero heat flux is prescribed at the boundary, it is not immediately obvious how that flux is split between the fluid and solid phases of the porous medium, and we are not aware of any previous discussion of this matter.

## 2. Analysis

The problem under investigation involves a relatively large number (five) of independent nondimensional parameters, and so in a pioneering study the obtaining of a reasonably simple analytic solution is a highly desirable goal. (This is especially so since the alternative numerical method typically leads to a problem which involves a small internal quantity (a temperature difference), and the traditional finite-difference formulation is ill-posed, and therefore, requires either a sophisticated numerical algorithm or a supercomputer for implementation.) Accordingly, we select a situation where an analytical solution is possible. We start by assuming that we have a fully developed hydrodynamic slug flow given by a Darcy model. An investigation of the more complicated flow given by a solution of the Brinkman equation is left until a later date. That means that here we have slip flow at the walls. In other words, we are supposing that the Darcy number  $Da = K/H^2$  (where  $K$  is the permeability and  $H$  is the channel half width) is small, so that the hydrodynamic boundary layer, whose thickness is of order  $Da^{1/2}$ , can be ignored.

We consider the case where the outside of the boundary slabs is maintained at a uniform constant temperature  $T_0$  and we neglect axial conduction both in the porous medium and in the boundary slabs. The

neglect of axial conduction within the porous medium is justified if the Péclet number is sufficiently large. The neglect of axial conduction in the boundary slabs is a restriction which we would like to remove at a later date. In the meantime, we suppose that the conductivity of the slabs is anisotropic, with a very small axial component, and consequently the conduction in the slabs is radial. In any case, the neglect of axial conduction in the slabs is consistent with the neglect of axial conduction in the porous medium, together with the uniform temperature imposed on the outside.

When the axial heat flux is zero, the temperature  $T'$  in the solid slabs is independent of the axial coordinate  $x$ . If  $T_0$  is the constant outside temperature, that at  $y = H + H'$ , then the solution of the heat conduction equation is

$$T' = T_0 + \beta(H + H' - y) \tag{1}$$

where  $\beta$  is constant. The temperature at the channel wall, at  $y = H$ , is thus,  $T_w = T_0 + \beta H'$ , and the wall heat flux is  $k'\beta$ , where  $k'$  is the slab conductivity. Let  $T_f$ ,  $T_s$  be, respectively, the temperature in the fluid, solid phases of the porous medium, and  $\phi$  be the porosity. Equating REV averages of the temperature and heat flux to the wall values we have, at  $y = H$ ,

$$\phi T_f + (1 - \phi)T_s = T_0 + \beta H' \tag{2}$$

$$\phi k_f (\partial T_f / \partial y) + (1 - \phi)k_s (\partial T_s / \partial y) = -k'\beta \tag{3}$$

where  $k_f$  and  $k_s$  are the fluid- and solid-phase conductivities, respectively.

Writing

$$\theta_f = T_f - T_0, \quad \theta_s = T_s - T_0 \tag{4}$$

and eliminating  $\beta$ , we have the boundary condition

$$\phi k_f (\partial \theta_f / \partial y) + (1 - \phi)k_s (\partial \theta_s / \partial y) = -(k'/H')\{\phi \theta_f + (1 - \phi)\theta_s\}, \tag{5}$$

at  $y = H$ .

Because the differential equations system, Eqs. (10) and (11) below, is of fourth-order, we need two boundary conditions at  $y = H$ . We postulate a uniformity principle that requires that the boundary condition holds for all values of the porosity  $\phi$ . Accordingly, we have

$$k_f (\partial \theta_f / \partial y) = -(k'/H')\theta_f \quad \text{and} \tag{6}$$

$$k_s (\partial \theta_s / \partial y) = -(k'/H')\theta_s, \quad \text{at } y = H.$$

(We note that when  $k' \rightarrow \infty$ , Eq. (6) reduces to  $\theta_f = 0$ ,  $\theta_s = 0$ , at  $y = H$ , corresponding to thermal equilibrium

at a perfectly conducting channel boundary, whereas when  $k' \rightarrow 0$ , Eq. (6) reduces to  $\partial\theta_f/\partial y = 0$ ,  $\partial\theta_s/\partial y = 0$ , at  $y = H$ , the expected conditions at an insulating channel boundary.)

We also have the symmetry conditions

$$\partial\theta_f/\partial y = 0, \quad \partial\theta_s/\partial y = 0 \quad \text{at } y = 0. \quad (7)$$

We assume that  $T_s$  and  $T_f$  are governed by the steady state heat transfer (energy) equations (Nield and Bejan [13], Eqs. (6.54) and (6.55))

$$(1 - \phi)\nabla \cdot (k_s \nabla T_s) + h_{fs}(T_f - T_s) = 0 \quad (8)$$

$$\phi \nabla \cdot (k_f \nabla T_f) + h_{fs}(T_s - T_f) = (\rho c_p)_f \mathbf{v} \cdot \nabla T_f. \quad (9)$$

Here  $h_{fs}$  is a fluid–solid heat transfer coefficient. For the case where the Darcy velocity  $\mathbf{v}$  has the uniform value  $U$  in the axial direction, for a homogeneous medium, and when axial conduction is neglected, Eqs. (8) and (9) reduce to

$$[(1 - \phi)k_s \partial^2/\partial y^2 - h_{fs}]\theta_s + h_{fs}\theta_f = 0 \quad (10)$$

$$h_{fs}\theta_s + [\phi k_f \partial^2/\partial y^2 - h_{fs} - U \partial/\partial x]\theta_f = 0. \quad (11)$$

Eqs. (10) and (11) must be solved subject to (6) and (7).

We now introduce dimensionless variables. We take  $H$  as length scale and  $T_{\text{ref}}$  as any convenient temperature scale. We will present our results in terms of a Nusselt number [defined in Eq. (33) below], the porosity  $\phi$ , and four other dimensionless parameters, namely a Biot number,  $Bi$ , a Péclet number,  $Pe$ , a porous medium conductivity ratio,  $k_r$ , and a solid–fluid heat exchange parameter,  $\eta$ , defined as follows:

$$Bi = k'H/k_{\text{eff}}H', \quad Pe = UH(\rho c_p)_f/k_r, \quad (12)$$

$$k_r = k_s/k_f, \quad \eta = h_{fs}H^2/k_{\text{eff}}$$

where  $k_{\text{eff}} = \phi k_f + (1 - \phi)k_s$ .

The parameter  $\eta$  is the reciprocal of the parameter  $N$  introduced by [12] but with  $k_s$  replaced by  $k_{\text{eff}}$ ; it is otherwise new to us. For convenience, we perform the algebra in terms of the parameters

$$\begin{aligned} L_f &= Bi[\phi + (1 - \phi)k_r], & L_s &= Bi[\phi + (1 - \phi)k_r]/k_r, \\ N_f &= \phi/Pe, & N_s &= (1 - \phi)k_r/Pe, \\ N_h &= \eta[\phi + (1 - \phi)k_r]/Pe. \end{aligned} \quad (13)$$

We let

$$y^* = y/H, \quad \theta_f^* = \theta_f/T_{\text{ref}}, \quad \theta_s^* = \theta_s/T_{\text{ref}} \quad (14)$$

substitute into Eqs. (10), (11), (6) and (7), and drop the asterisks. We then get

$$[N_s \partial^2/\partial y^2 - N_h]\theta_s + N_h\theta_f = 0 \quad (15)$$

$$N_h\theta_s + [N_f \partial^2/\partial y^2 - N_h - \partial/\partial x]\theta_f = 0 \quad (16)$$

$$\begin{aligned} \partial\theta_f/\partial y + L_f\theta_f = 0 \quad \text{and} \quad \partial\theta_s/\partial y + L_s\theta_s = 0 \\ \text{at } y = 1 \end{aligned} \quad (17)$$

$$\partial\theta_f/\partial y = 0, \quad \partial\theta_s/\partial y = 0 \quad \text{at } y = 0. \quad (18)$$

The homogeneous system of equations (15)–(18) can be solved using the method of separation of variables. Letting

$$\theta_f = \Theta_f(y) e^{\lambda x}, \quad \theta_s = \Theta_s(y) e^{\lambda x} \quad (19)$$

and denoting  $d/dy$  by  $D$ , we get

$$(N_s D^2 - N_h)\Theta_s + N_h\Theta_f = 0 \quad (20)$$

$$N_h\Theta_s + (N_f D^2 - \lambda - N_h)\Theta_f = 0 \quad (21)$$

$$D\Theta_f + L_f\Theta_f = 0, \quad D\Theta_s + L_s\Theta_s = 0, \quad \text{at } y = 1 \quad (22)$$

$$D\Theta_f = 0, \quad D\Theta_s = 0, \quad \text{at } y = 0. \quad (23)$$

Eliminating  $\Theta_s$ , we get

$$\{(N_s D^2 - N_h)(N_f D^2 - N_h - \lambda) - N_h^2\}\Theta_f = 0 \quad (24)$$

$$\begin{aligned} D\Theta_f + L_f\Theta_f = 0, \\ (D + L_s)(N_f D^2 - N_h - \lambda)\Theta_f = 0, \quad \text{at } y = 1. \end{aligned} \quad (25)$$

The solution of (24) subject to the symmetry requirement (23) is

$$\Theta_f = A \cos s_1 y + B \cosh s_2 y \quad (26)$$

where

$$\begin{aligned} s_1 = (\{-[N_h(N_f + N_s) + \lambda N_s] + \{[N_h(N_f + N_s) \\ + \lambda N_s]^2 - 4\lambda N_f N_s N_h\}^{1/2}\}/2N_f N_s)^{1/2} \end{aligned} \quad (27)$$

$$\begin{aligned} s_2 = (\{[N_h(N_f + N_s) + \lambda N_s] + \{[N_h(N_f + N_s) \\ + \lambda N_s]^2 - 4\lambda N_f N_s N_h\}^{1/2}\}/2N_f N_s)^{1/2} \end{aligned} \quad (28)$$

and  $A$  and  $B$  are constant. Substituting into Eq. (25), and eliminating  $A$  and  $B$ , we get the eigenvalue

equation for  $\lambda$ , which can be written in the form

$$\begin{aligned} (N_f s_1^2 + N_h + \lambda)(s_1 \tan s_1 - L_s)(s_2 \tanh s_2 \\ + L_f) = (-N_f s_2^2 + N_h + \lambda)(s_2 \tanh s_2 \\ + L_s)(s_1 \tan s_1 - L_f). \end{aligned} \quad (29)$$

The significant eigenvalue is the negative root of smallest magnitude. The corresponding values of  $s_1$  and  $s_2$  are real. The eigenvector is obtained from

The calculation of  $Nu$  is now straightforward. The factor  $T_{\text{ref}} e^{\lambda x/U}$  that appears in both  $q''$  and  $T_{\text{b,eff}} - T_0$  now cancels in the calculation. One can use (30), together with the polynomial equations satisfied by  $s_1$  and  $s_2$ , and the relations

$$k_{\text{eff}} = \phi k_f + (1 - \phi)k_s = UH(\rho c_p)_f(N_f + N_s)$$

$$\phi k_f = UH(\rho c_p)_f N_f$$

$$(1 - \phi)k_s = UH(\rho c_p)_f N_s$$

to obtain the formula for the Nusselt number in the form

$$Nu = \frac{(L_f C_2 + s_2 S_2) s_1 S_1 \left[ N_f + \frac{N_s N_h}{N_h + N_s s_1^2} \right] + (L_f C_1 - s_1 S_1) s_2 S_2 \left[ N_f + \frac{N_s N_h}{N_h - N_s s_2^2} \right]}{\left( \frac{N_f + N_s}{2} \right) \left\{ \frac{(L_f C_2 + s_2 S_2) S_1}{s_1} \left[ \phi + \frac{(1 - \phi) N_h}{N_h + N_s s_1^2} \right] - \frac{(L_f C_1 - s_1 S_1) S_2}{s_2} \left[ \phi + \frac{(1 - \phi) N_h}{N_h - N_s s_2^2} \right] \right\}} \quad (36)$$

$$\begin{aligned} B/A = (s_1 \sin s_1 - L_f \cos s_1)/(s_2 \sinh s_2 \\ + L_f \cosh s_2) \end{aligned} \quad (30)$$

and from (21) one obtains

$$\begin{aligned} \Theta_s = N_h^{-1} \{ (N_f s_1^2 + N_h + \lambda) A \cos s_1 y \\ + (-N_f s_2^2 + N_h + \lambda) B \cosh s_2 y \}. \end{aligned} \quad (31)$$

The heat flux at the channel wall can then be determined from

$$q'' = \phi k_f (\partial T_f / \partial y)_{y=H} + (1 - \phi) (\partial T_s / \partial y)_{y=H}. \quad (32)$$

The Nusselt number is defined by

$$Nu = 2Hh/k_{\text{eff}} \quad (33)$$

where, in turn,

$$h = q'' / (T_0 - T_{\text{b,eff}}) \quad (34)$$

where the effective bulk temperature

$$\begin{aligned} T_{\text{b,eff}} = \frac{1}{U} \int_0^H u \{ \phi T_f + (1 - \phi) T_s \} dy \\ = \int_0^H \{ \phi T_f + (1 - \phi) T_s \} dy. \end{aligned} \quad (35)$$

(The reader should note that in [12] the heat transfer coefficient was defined less appropriately, in terms of the fluid bulk temperature rather than the REV average of fluid and solid bulk temperatures.)

where  $C_1 = \cos s_1$ ,  $C_2 = \cosh s_2$ ,  $S_1 = \sin s_1$ ,  $S_2 = \sinh s_2$ .

The solution for the case  $k_f = k_s$  is especially simple. One finds that in this case

$$\Theta_f = A \cos \mu y \quad (37)$$

and

$$Nu = 2\mu^2 \quad (38)$$

where  $\mu$  is the smallest positive root of the equation

$$x \tan x = Bi. \quad (39)$$

Some results based on Eqs. (38) and (39) are given in Table 1.

Table 1  
Values of Nusselt number vs. Biot number, calculated from Eqs. (38) and (39), for the case of thermal equilibrium

$Bi$	$\mu$	$Nu$
0	0	0
0.01	0.09983	0.01993
0.05	0.22176	0.09835
0.1	0.31105	0.1935
0.5	0.65327	0.8535
1	0.86033	1.4803
5	1.31384	3.4524
10	1.42887	4.0833
50	1.54001	4.7433
100	1.55525	4.8376
500	1.56766	4.9151
1000	1.56923	4.9250
$\infty$	1.57080( $\pi/2$ )	4.9348( $\pi^2/2$ )

The eigenfunction solution so far obtained for our parabolic differential equation system contains a multiplicative factor whose determination requires that an upstream ('initial') condition be specified. The eigenfunction is such that its shape (expressed by the dependence on  $y$ ) does not evolve with distance downstream but the amplitude (expressed by the dependence on  $x$ ) decays exponentially as  $x$  increases.

After some algebra, we can show that the formula for  $Nu$ , Eqs. (38) and (39), is also applicable in each of the limiting cases  $\phi \rightarrow 0$ ,  $\phi \rightarrow 1$ ,  $\eta \rightarrow \infty$ . The reader will note that these cases (together with the case  $k_r = 1$ ) are those corresponding to local thermal equilibrium. At an intermediate state of the algebra, we find that  $s_2$  tends to infinity for each of these limiting cases.

When  $\eta \rightarrow 0$  the situation is different. In this case one finds that, in the limit

$$Nu = 2\mu^2 / \{\phi + (1 - \phi)k_r\} \quad (40)$$

where now  $\mu$  is the smallest positive root of

$$x \tan x = Bi \{\phi + (1 - \phi)k_r\}. \quad (41)$$

### 3. Results

An immediate dramatic result is that the Nusselt number is not affected by the value of the Péclet number. This can be seen from the expression (36) for  $Nu$ , which is homogeneous of degree zero in the parameters  $N_f$ ,  $N_s$ , and  $N_h$ , each of which is inversely proportional to  $Pe$ , and the expression is otherwise independent of  $Pe$ . The rate of decay in the axial direction, expressed by  $\lambda$ , is also inversely proportional to  $Pe$ . (We checked these conclusions by computation.)

The major effect of varying the Biot number is illustrated by Table 1. In the absence of thermal nonequilibrium, the effect of finite resistance of the boundary slab is to decrease the heat transfer, and the Nusselt number decreases as the Biot number decreases, as shown in the table.

The Biot number also interacts with the exchange parameter as illustrated by Figs. 2 and 3. The case of large  $\eta$  corresponds to thermal equilibrium. As  $\eta$  decreases from large values, the Nusselt number decreases if  $k_r > 1$ , i.e. if the solid conductivity exceeds the fluid conductivity in the porous medium. Then the effect of the LTNE is to decrease the amount of heat transfer into the porous medium, and the change with variation of  $\eta$  is monotonic. The effect of LTNE becomes less as the Biot number decreases. It appears that the increased thermal resistance of the boundary slabs interferes with the LTNE effect.

On the other hand, if  $k_r < 1$  then the LTNE effect is more ambiguous. The Nusselt number decreases

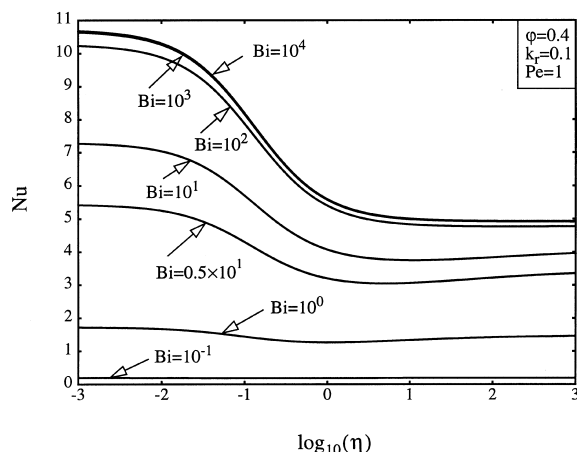


Fig. 2. Plots of Nusselt number vs. logarithm of exchange parameter, for various values of Biot number, for the case porosity=0.4 (a typical value), conductivity ratio=0.1 (illustrating the case of solid conductivity less than fluid conductivity) and Péclet number=1 (in fact, these results are independent of Péclet number).

slightly as  $\eta$  decreases from large values, and goes through a minimum before increasing substantially as  $\eta$  decreases to zero. For the case of large Biot numbers, the minimum is barely, if at all, observable.

The effect of variation of a Nusselt number with porosity, with other parameters fixed, is illustrated by Figs. 4 and 5. When  $k_r = 1$ ,  $Nu$  is independent of  $\phi$ . When  $k_r > 1$ , the shape of  $Nu$  vs.  $\phi$  curve is like that of a hanging nonuniform chain suspended between supports of equal height. We have seen that when  $\phi = 0$  or  $\phi = 1$  there is thermal equilibrium, and so that amount of dip of the curve below the end values is a measure

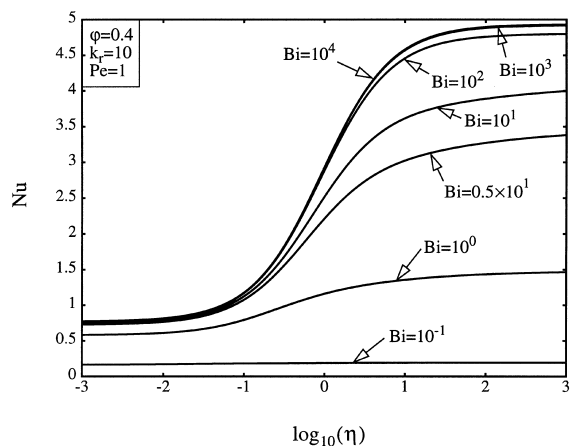


Fig. 3. As in Fig. 2, but for conductivity ratio = 10 (illustrating the case of solid conductivity greater than fluid conductivity).

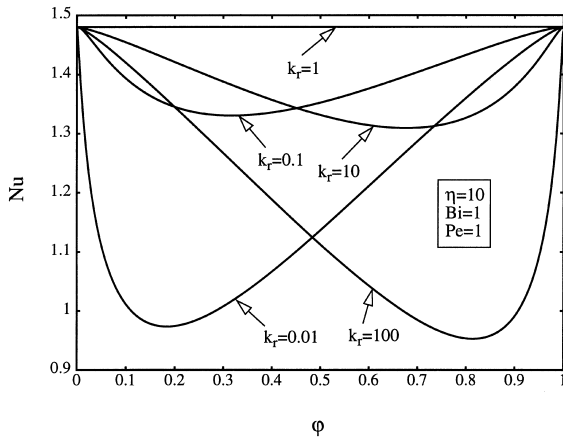


Fig. 4. Plots of Nusselt number vs. porosity, for various values of the conductivity ratio, for the case Biot number=1, exchange parameter = 10.

of the effect of LTNE. When  $k_r < 1$ , the situation is more complicated. As we have seen in Fig. 2, there is a domain of  $(Bi, \eta)$  values ( $Bi$  sufficiently small,  $\eta$  sufficiently large), for which the LTNE effect leads to a reduction in  $Nu$ , and for these the corresponding  $Nu$  vs.  $\phi$  curves are of the normal hanging chain type. For the complementary  $(Bi, \eta)$  domain, for which the LTNE effect leads to an increase in  $Nu$ , the  $Nu$  vs.  $\eta$  curves are now S-shaped, with a depression for large  $\phi$  values but an elevation for small  $\phi$  values, relative to the thermal equilibrium values. The computations become more difficult as  $k_r$  becomes small, and that is why in Fig. 5 the smallest value for  $k_r$  used is 0.05 rather than 0.01. The solution becomes singular at  $\phi = 0$  when  $k_r$  tends to zero.

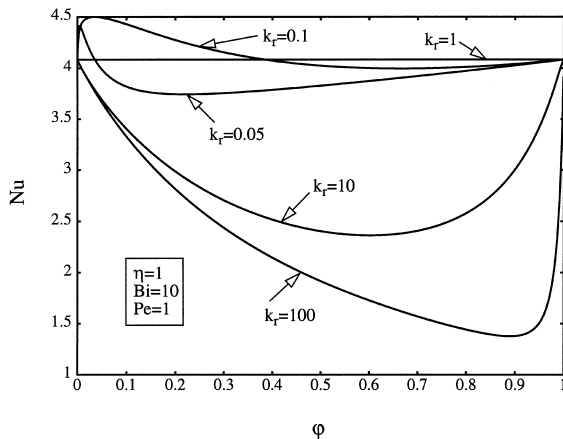


Fig. 5. As in Fig. 4, but for the case Biot number=10, exchange parameter = 1.

## 4. Discussion

### 4.1. Boundary conditions

In the course of the above analysis we produced two boundary conditions from one [see Eqs. (5) and (6)] by applying a uniformity principle. Before adopting this principle, we briefly considered an alternative approach to deriving a second boundary condition, namely one based on the exchange coefficient  $h_{fs}$ , but we rejected this approach for two reasons. First, we believe that the formulation involving  $h_{fs}$  models a convective effect, one involving relative motion of fluid past solid particles, and although with a Darcy model we have slip at the wall, in general there will be no slip, and therefore, no relative motion, at the wall. Second, on philosophic grounds, applying Occam's razor, we prefer a simple, elegant formulation. The uniformity principle allows the treatment of the fluid and solid phases in an even-handed fashion. With its aid we can now resolve the question raised in the introduction to this paper. In the case where uniform heat flux is applied at the boundary of the porous medium, we require that the heat flux is truly uniform with respect to the fluid and solid phases. This means that the boundary heat flux into an REV is to be split between the fluid and solid phases in the ratio  $\phi:(1-\phi)$ , so that the rate of heat transfer per unit area is uniform.

### 4.2. Limitations of the model

Perhaps the major limitation of our model is the assumption that axial heat transfer is negligible in the boundary slabs as well as in the porous medium. We expect that the presence of axial conduction will interfere with the transfer of heat from the boundary slabs to the porous medium. If that is correct, then the Nusselt number values reported in this paper are upper bounds on the true values.

We have reported results for all values of the heat exchange parameter  $\eta$  from zero to infinity, but we recognize that in most practical circumstances the value of  $\eta$  will be large. Thus, the increase of  $Nu$  above the corresponding thermal equilibrium value, which occurs for small  $\eta$  and  $k_r < 1$ , is exceptional. It occurs as a result of heat flow being channeled into the highly conducting fluid phase of the medium at the expense of the poorly conducting solid phase.

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